Thomas E. : Friend and my *"Passepartout* for French TCS B" since 35 Years

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How we met

Via P.-L. Curien whom I knew from Marktoberdorf Summer School in early 80s. In 1987 at some meeting he introduced me to Thomas E. since for our Theses we were both working on categorical semantics of the *Calculus of Constructions* (as various other people those days).

Pierre-Louis invited me to spend 3 weeks at ENS rue d'Ulm in early summer 1988 which gave us the opportunity to compare and discuss our approaches at numerous informal seminars.

I was using Cartmell's **contextual categories** and concentrated on correctness and completeness results.
We both were using realizability models as standard models which were in the air those days anyway.
Thomas' approach was based on *Grothendieck fibrations* which are certainly more elegant but also a bit too general (it suffices to restrict to full subfibrations of the fundamental fibration of the base category).

Sequentiality, Full Abstraction etc.

My visit at ENS in 1988 opened up to me a new world and I met various French colleagues the first time.

The full abstraction problem for PCF was quite topical those days and starting with my visit in 1988 I learnt about Pierre-Louis' approach two sequentiality.
His **sequential algorithms** on **concrete data structures** in retrospect were the first games model.
But fully abstract not for PCF itself but rather for PCF extended by a catch construct allowing one to compute sequentiality indices and thus strategies (as later worked out in detail by Cartwright, Curien and Felleisen).

Strong Stability

But the *holy grale* was to give an extensional characterization of the fully abstract model.

For PCF proper this can be done via Kripke logical relations but even for booleans one has to add an infinite amount of structure which has to be preserved by morphisms.

But the extensional quotient of the category $\mathbf{SA}$ of sequential algorithms can be characterized as the category $\mathbf{SS}$ of **strongly stable** functions between so-called **hypercoherences**, an ingeniously chosen category of **hypergraphs** generalising **coherence spaces**, i.e. undirected graphs in the combinatorialist's sense.

Hypercoherences and strongly stable functions have a linear refinement providing a model of classical linear logic.

Further New Models of Linear Logic

One might expect that Linear Logic has models in categories of vector spaces (of some restricted kind) and (certain) linear maps between them. The restrictions are necessary since Banach spaces are typically not reflexive and thus don't model **classical** linear logic.

The first functional analytic model was given by so called **Köthe spaces** having a basis of open subspaces.
These turned out as as induced by so-called **finiteness spaces** generalising coherence spaces.

The latter do not model general recursion but do model nondeterminism!

Relational Model and Scott Model of LL

Sets and relations provide a model of linear logic when interpreting $!X$ as finite **multi**(sub)sets of X.
Complete prime algebraic lattices and linear Scott continuous functions between them give rise to the so-called **Scott model** of linear logic.
As shown by Thomas E. the latter appears as **extensional collapse** of the above relational model.

NB There is nothing like $\mathbb{N}_\perp$ in Thomas' Scott model.
It is more like a continuation model where objects are powers of Σ (Sierpiński space) and morphisms are arbitrary Scott continuous maps.

Differential Models

Thomas observed that the finiteness model hosts for every object X not only the deriliction morphism $!X \multimap X$ but also an antideriliction morphism $X \multimap !X$.
This allows one to asscociate with every morphism $f : !X \multimap Y$ its **derivative** $Df : !X \otimes X \multimap Y$.

Inspired by this model Thomas develops (with L. Regnier) **differential** λ-calculus and linear logic.
In such calculi differentiation allows one to compute for functions $f : X \to Y$ its **Taylor expansion**, the (infinite) product of all its *multi-ary linearizations* based on the idea that from $!X \cong \coprod_{n \in \omega} X^{\otimes n}$ we obtain $[X \to Y] = [!X \multimap Y] \cong \prod_{n \in \omega} [X^{\otimes n} \multimap Y]$.

Probabilistic Functional Programming 1

Based on ideas by Girard Thomas developed (with V. Danos) the **Probabilistic Coherence Space** model of LL. Cliques are not just subsets of the web $|X|$ but rather functions $|X| \to [0,\infty[$ where $x \perp y$ iff $\sum_{i \in I} x_i y_i \leq 1$. A probabilistic coherence space then is a biorthogonally closed subset of $[0,\infty[^{|X|}$.

The coKleisli category provides a model of **Probabilistic PCF** (with a **let**-construct restricted to integers) which in joint work with Pagani and Tasson he has shown to be fully abstract!
The proof is not based on a definability result for a dense subset but rather on the fact that analytic functions are equal whenever they coincide on a nonempty open subset.

Generalizing probabilistic coherence spaces to **cones** allows one to have arbitrary spaces of finite measures as types.
Moreover one gets integration and differentiation as morphisms.

Probabilistic Functional Programming 2

Thus, one obtains a model of **Probabilistic Functional Programming** as used in applied fields like **data mining**, **neural networks** etc.

Maybe this opens up the possibility to make **"denotational semantics great again"** – at least to some extent.

What is lacking in my eyes is the relation to computability in the sense of TTE or **function realizability** where probabilistic aspects are added via **probabilistic powerdomain** or **Daniell-Stone integration** (measures on X are positive linear functionals from $\mathbb{R}^X$ to $\mathbb{R}$) as a particular **computational effect**.

What Next?

Thomas has forcefully demonstrated that Denotational Semantics is alive and can be a driving force in developing new notions also relevant for applications.

61 is not very old and so we can be curious what will come next!